Weibull-based Benchmarks for Bin Packing

CP'12

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Outline

1. Motivation
2. Weibull Approach
3. Fitting to Real-World Instances
4. Experimental Setup
5. Systematic Solving
6. Heuristic Solving
7. Summary and Perspectives

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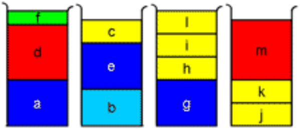
Bin Packing Problem

The uni-dimensional Bin Packing Problem (BPP) is a classical combinatorial optimization problem.



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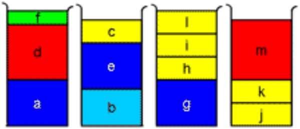

- Set I of n Items defined by an integer size $s_i \geq 0$ $\forall i \in \{1, \ldots, n\}$
- Set J of m Bins with a positive capacity C

Goal : Finding an assignement for each item to a single bin without breaking the capacity constraints, such that the number of used bins is minimized.

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- Timetabling
- Scheduling
- Stock cutting

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Literature contains many different approaches to solve it:

- Genetic Algorithms *(Falkenauer 1996)*
- Operations Research Methods *(Cambazard and O'Sullivan 2010)*
- Satisfiability Techniques *(Grandcolas and Pinto 2010)*
- Constraint Programming *(Dupuis et al. 2010, Shaw 2004)*
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Most of these approaches rely in their own set of benchmarks

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A benchmark generator allowing us to develop realistic setups for measuring the performance of different solving methods.

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Two main characteristics:

- It should fit well existing real-world BPP instances.
- It should be precise enough to detailed control the instances being generated (to perform very controlled experiments).

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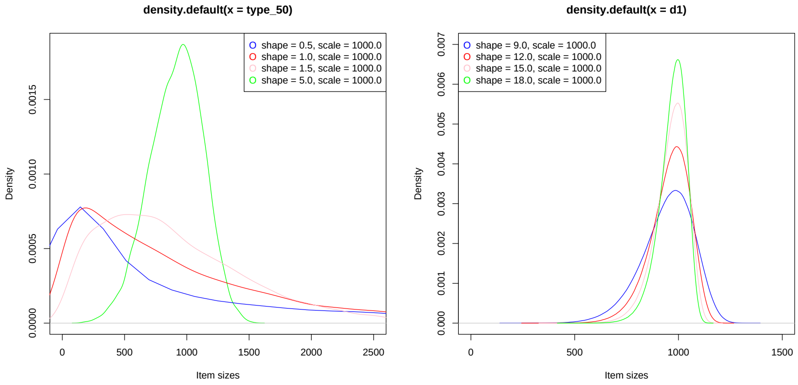
$$f(x;\lambda,k) = \begin{cases} \frac{k}{\lambda} \cdot \left(\frac{x}{\lambda}\right)^{k-1} \cdot e^{-(x/\lambda)^k} & x \geq 0, \\ 0, & \text{otherwise} \end{cases}$$

Weibull Approach

Weibull distribution for different values of the shape (scale fixed)



(a) Shape $k \in \{0.5, 1.0, 1.5, 5.0\}$ (b) Shape $k \in \{9.0, 12.0, 15.0, 18.0\}$

Figure: Weibull distributions with fixed scale. $\lambda = 1000$

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Both problems can be studied as extensions of the uni-dimensional BPP

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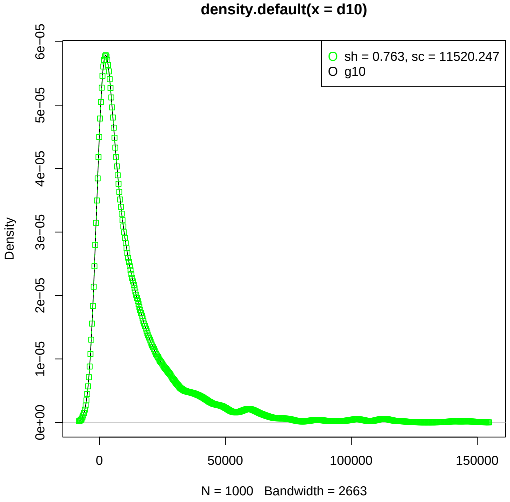
5. Statistical goodness-of-fit tests

4. Observation: (Wessa Online Service)



Fitting to real-world instances

5. Statistical tests: (Kolmogorov-Smirnov and χ^2)

		Weibull Best-fit		KS test	χ^2 test		
Set	Instance	shape	scale	p-value	#(cat)	lbTail	p-value
ETT	Nott	1.044	43.270	**0.7864**	7	100	**0.059**
ETT	MelA	0.946	109.214	**0.091**	10	427	**0.073**
ETT	MelB	0.951	117.158	**0.079**	5	47	**0.051**
ETT	Cars	1.052	85.438	0.037	18	53	**0.109**
ETT	hec	1.139	138.362	**0.436**	10	293	**0.204**
ETT	yor	1.421	37.049	**0.062**	7	117	**0.068**
RAODEF	$a1_3^2$	0.447	104,346.70	0.005	30	163,000	**0.105**
RAODEF	$a1_3^3$	0.549	88,267.85	0.001	15	54,800	**0.068**
RAODEF	$a2_1^5$	0.562	67,029.83	0.000	30	470,000	**0.768**
RAODEF	$a2_4^4$	0.334	103,228.30	0.001	30	500,000	**0.051**
RAODEF	b_6^3	0.725	40,469.74	0.000	20	185,000	**0.060**
RAODEF	b_3^5	0.454	91,563.28	0.000	30	140,000	**0.088**

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Number of Items	N $= 100$
Weibull Scale	$\lambda = 1000$
Weibull Shape	$k \in \{0.1, 0.2, \ldots, 19.9\}$
Bin Capacity Factor	$C \in \{1.0, 1.1, \ldots, 2.0\}$

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Analysis will be held for both systematic and heuristic solving.

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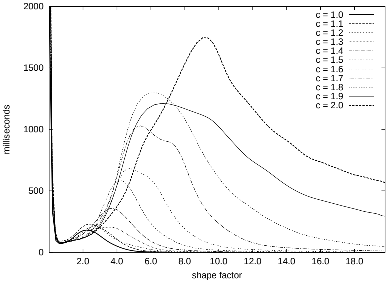
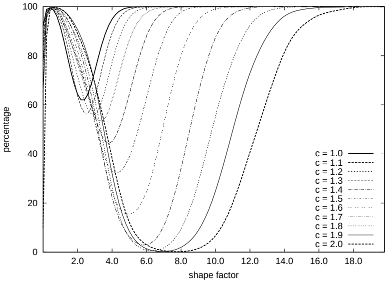
- Complete Decreasing Best Fit Search (Gent and Walsh 1997)
- Timeout

Considering solving time and the percentage of instances solved.



(a) Average running time for instances that did not timeout



(b) Percentage of instances solved within the timeout

Figure: Shape $k \in \{0.1, 0.2, \ldots, 19.9\}$

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- Number of bins increases as shape parameter increases

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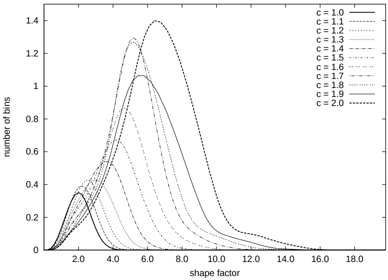
Heuristic Solving

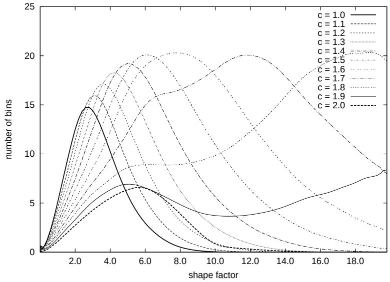
Four different strategies (Rieck implementation)

- MaxRest
- FirstFit
- BestFit
- NextFit

Considering solving quality w.r.t the optimal solution of the CP method



(a) Difference in the average number of bins in solutions found using MaxRest



(b) Difference in the average number of bins in solutions found using NextFit

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- Whereas the greediness of the NextFit heuristic does not pay off, the more considered reasoning used by the MaxRest, FirstFit and BestFit heuristics does.
- For them, the quality of the solutions obtained follows the difficulty for the CP method

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- We have showed that our approach can very accurately model real-world bin packing problems.
- We have presented an empirical analysis of both systematic search and heuristic methods for BPP based on a large benchmark suite generated using our approach, showing a variety of interesting behaviours that are otherwise difficult to observe systematically.

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- Use our three parameters model {Shape, Scale, Bin capacity} as a a basis for tuning BPP methods and generating portfolio-based BPP solvers relying on these parameters for learning their best configuration.
- Extend the model to produce benchmark generators for a variety of other important problems.

End

Thank you for your attention